

# Qualitative Model-Based Diagnosis Using Possibility Theory\*

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## Abstract

The potential for the use of possibility theory in the qualitative model-based diagnosis of spacecraft systems is described. The first sections of the paper briefly introduce the Model-Based Diagnostic (MBD) approach to spacecraft fault diagnosis; Qualitative Modeling (QM) methodologies; and the concepts of possibilistic modeling in the context of Generalized Information Theory (GIT). Then the necessary conditions for the applicability of possibilistic methods to qualitative MBD, and a number of potential directions for such an application, are described.

**Possibility theory** is being developed as an alternative to traditional theories of uncertainty. While possibility is logically independent of probability theory, they are related: both arise in **Dempster-Shafer evidence theory** as **fuzzy measures** defined on **random sets**; and their distributions are **fuzzy sets**. Together these fields comprise the new field of **Generalized Information Theory**.

**Possibilistic processes**, which generalize interval analysis, are based on a set of partially overlapping intervals, resulting in non-additive weights on a set of alternatives. Thus they are suitable for **qualitative modeling** methods, which inherently require loose representations of uncertainty, and typically involve **interval analysis**.

Qualitative methods are appropriate for modeling complex systems, such as spacecraft, where the interaction among the large number of parts and varying environmental conditions results in the possibility of unpredictable behavior and long-run departure from established steady-state domains. Therefore it is hypothesized that possibilistic methods may be useful for qualitative model-based diagnosis and trend analysis of spacecraft systems.

## 1 Model-Based Diagnosis

The **model-based** approach to systems **diagnosis** (MBD) [17] is based on the premise that knowledge about the internal structure of a system can be useful in diagnosing its failure. In MBD, a software model of the system, given inputs from the real system, generates and tests various failure hypotheses.

A typical MBD approach (derived from some of the standard literature [2, 7, 16]) to diagnosing a spacecraft (here described as some internal **system** whose **sensor** measurements output to a **telemetry** stream) is shown in Fig. 1.

An **alarm** is a report that some observed system attributes have departed from nominal, and entered error, conditions, usually by exceeding some threshold values; a **prediction** is a report that some system attributes should be in certain states; a **fault hypothesis** is a list of system components which may

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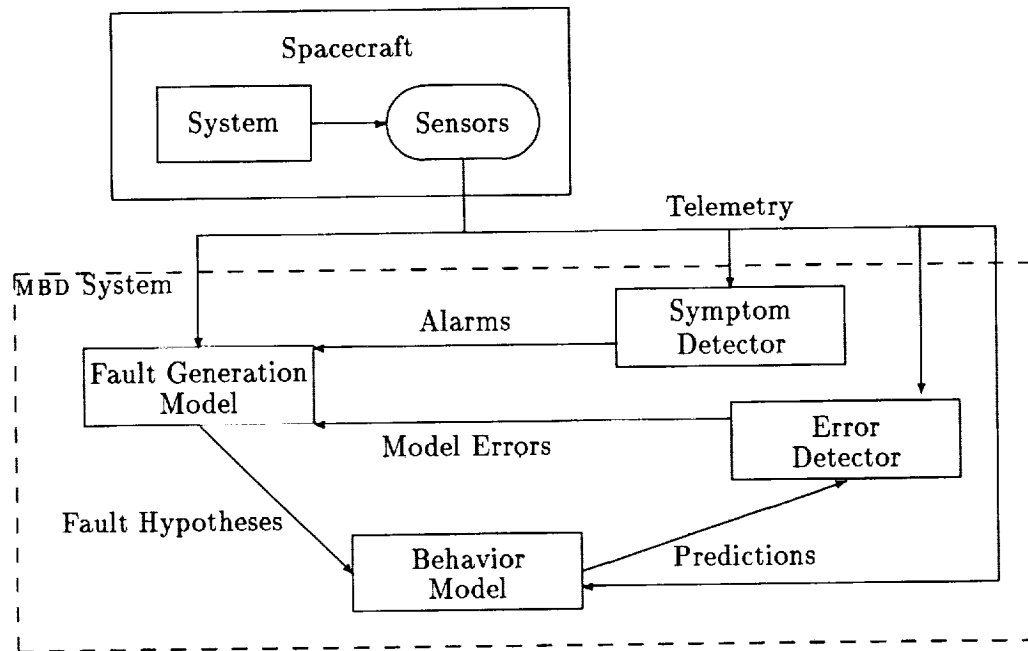


Figure 1: A typical model-based diagnostic system.

have failed; and an **error** is a report of a discrepancy between predicted and measured system attribute states.

The overall MBD system then involves two distinct spacecraft models. The **fault generation model** (FGM) takes inputs from telemetry, alarms, and errors, and either produces anew, or modifies existing, fault hypotheses. The **behavior model** takes inputs from telemetry and fault hypotheses, and outputs predictions. These are then corroborated against telemetry to produce errors. The fault hypotheses act to modify the behavior model so that it predicts system behavior as if the hypothetical system components had actually failed.

Both models can be difficult to construct, typically involving delicate tradeoffs among accuracy, precision, and tractability. But the FGM, as the heart of the MBD approach, is particularly complex and involved. The FGM could be, for example, an inversion of the behavior model (as for Dvorak and Kuipers [7]) or a decision tree (as for Shen and Leitch [31]). Through backwards reasoning a variety of subsets of components can be identified, any of which are consistent with the given telemetry and alarms.

**Filtering** is the process by which error output is used to prune the set of fault hypotheses. If the prediction of the behavior model as modified by a particular fault hypothesis produces errors, then that fault hypothesis is not retained. As the system is monitored over time, further observations narrow the class of fault hypotheses. Achieving the null set indicates model insufficiency. But if the overall MBD system stabilizes to a non-empty set of fault-hypotheses, then these are advanced as possible causes of the failure.

## 2 Qualitative Modeling

**Qualitative modeling** (QM), usually considered a part of artificial intelligence, can be broadly described as the attempt to deliberately model systems at a high level of abstraction from the actual systems

themselves. Of course this approach produces models which are less precise than they might be, but with the tradeoff of potentially greater tractability and accuracy (the less you say, the better your chance of being right).

QM methods can be useful when there is only a poor model of the original system, or when there are missing or incomplete data. This can happen when systems are incompletely specified, when they have parameters or states which aren't always known with certainty, or when complexity makes detailed prediction difficult. For more information about QM in general, see the anthologies edited by Bobrow [1], and Fishwick and Luker [10], and survey articles by Fishwick [8] and Guariso, Rizzoli and Werthner [14].

There are a variety of broad approaches within QM, which come under the names of naive physics, qualitative physics, qualitative simulation, qualitative reasoning, qualitative dynamics, etc. There are also a number of specific methods, including bond graphs, causal loop modeling, natural language modeling, "lumped" state space models, and inductive approaches.

In this paper the QM methods of most interest are those which use uncertainty distributions on state variables, and mixed interval- and point-valued dynamical systems. In models using uncertainty distribution methods the uncertainty about some attribute is represented mathematically by weights on all possible values. The set of weights, as a distribution, acts as a meta-state in the space of all possible distributions, and functional equations relating these meta-states produce predictions about the distribution meta-state at future times. Models using probability distributions are familiar as Markov processes and other kinds of stochastic models, and these have correlates in possibility theory (see Sec. 3.2). These methods are actually semi-qualitative, since the numerical representation of the distribution adds a quantitative component.

In an interval-valued dynamical modeling system like QSIM [28], a precise point-valued dynamical system of differential or difference equations is replaced by a homomorphic interval-valued process. Typically qualitative variables are identified within certain intervals, some relatively unconstrained (for example  $x \in [0, \infty)$ ), and some constrained by landmark values (for example  $x \in [x_{\min}, x_{\max}]$ ).

Qualitative variables are then generally related in three ways:

**Functional:** For example, if  $y = M^-(x)$  then  $y$  is a monotonically decreasing function of  $x$ , so that if  $x \in [0, x_{\max})$  then  $y \in (-\infty, 0]$  or  $y \in (M^-(x_{\max}), 0]$ .

**Arithmetic:** Standard mathematical operations can also be represented qualitatively, for example if  $x \in [0, x_{\max}]$  and  $y \in (-\infty, 0]$ , then  $xy \in (-\infty, 0]$ , but  $x + y$  is unknown.

**Dynamic:** Change of state is represented by qualitative magnitude and direction. Qualitative differential relations link directions with magnitudes, for example given  $y = dx/dt$ , then

$$x \text{ increasing} \rightarrow y \in (0, \infty), \quad x \text{ decreasing} \rightarrow y \in (-\infty, 0).$$

Of course, determinative results may not be available in such a qualitative model. For example, we saw above that  $x + y$  could be any value in  $(-\infty, \infty)$ . Similarly, the existence of landmark values leads to uncertainty as to whether a landmark has been crossed. To account for each possibility, two alternatives must be branched off. Therefore in general, QM systems have a tree of possible system behaviors, and external factors (heuristics or other constraints) may be required to prune that tree.

QM has been applied to MBD to produce qualitative model-based diagnostic systems. For example, in the approach of Dvorak and Kuipers [7], model predictions are intervals of possible system state values. Stochastic methods, for example Bayesian networks [12] and Markov processes [13], have been

used extensively in MBD applications. And recently Shen and Leitch [31, 32] have advanced the FUSIM method for qualitative MBD which uses fuzzy arithmetic (see Sec. 4.2).

### 3 Possibility Theory

Possibility theory was originally developed in the context of fuzzy systems theory [34], and was thus related to the kinds of cognitive modeling that fuzzy sets are usually used for. More recently, possibility theory is being developed as a new form of mathematical information theory complementing probability theory in the context of Generalized Information Theory (GIT). The author is developing possibility theory based on consistent random sets, and also an empirical semantics for possibility, including possibilistic measurement procedures and applications to the modeling of physical systems.

#### 3.1 Possibilistic Mathematics in GIT

Table 3.1 summarizes the primary formulae of probability and possibility theory in the context of GIT and random set theory. These are only briefly explained here; see [5, 24, 26, 27] for more information.

Given a finite universe  $\Omega := \{\omega_i\}, 1 \leq i \leq n$ , the function  $m: 2^\Omega \mapsto [0, 1]$  is an **evidence function** (otherwise known as a **basic probability assignment**) when  $m(\emptyset) = 0$  and  $\sum_{A \subseteq \Omega} m(A) = 1$ . Denote a random set generated from an evidence function as  $\mathcal{S} := \{\langle A_j, m_j \rangle : m_j > 0\}$ , where  $\langle \cdot \rangle$  is a vector,  $A_j \subseteq \Omega, m_j := m(A_j)$ , and  $1 \leq j \leq N := |\mathcal{S}| \leq 2^n - 1$ . Denote the **focal set** as  $\mathcal{F} := \{A_j : m_j > 0\}$  with **core**  $\mathbf{C}(\mathcal{F}) := \bigcap_{A_j \in \mathcal{F}} A_j$  and **support**  $\mathbf{U}(\mathcal{F}) := \bigcup_{A_j \in \mathcal{F}} A_j$ .

The **plausibility** and **belief** measures on  $\forall A \subseteq \Omega$  are

$$\text{Pl}(A) := \sum_{A_j \cap A \neq \emptyset} m_j, \quad \text{Bel}(A) := \sum_{A_j \subseteq A} m_j.$$

Since they are dual, in that  $\text{Bel}(A) = 1 - \text{Pl}(\bar{A})$ , in general only plausibility will be considered below. The **plausibility assignment** (otherwise known as the **one-point coverage function**) of  $\mathcal{S}$  is  $\vec{\text{Pl}} = \langle \text{Pl}_i \rangle := \langle \text{Pl}(\{\omega_i\}) \rangle$ , where

$$\text{Pl}_i := \sum_{A_j \ni \omega_i} m_j. \quad (1)$$

$\vec{\text{Pl}}$  is a **fuzzy subset** of  $\Omega$  that can be mapped to an equivalence class of random sets on  $\Omega$ .

Under certain conditions the evidence values  $m_j$  and the plausibility assignment values  $\text{Pl}_i$  are mutually determining. Then  $N \leq n$ , and  $\vec{\text{Pl}}$  is a **distribution** of  $\mathcal{S}$ . When  $N = n$  (there are exactly as many focal elements as there are elements of the universe), then the indices  $j$  on the focal elements  $A_j$ , and the  $i$  on the universe elements  $\omega_i$  are equivalent, and it may be useful to use one or the other interchangeably. This is the case in the two rightmost columns of the table.

When  $\forall A_j \in \mathcal{F}, |A_j| = 1$ , then  $\mathcal{S}$  is **specific**, and  $\text{Pr}(A) := \text{Pl}(A) = \text{Bel}(A)$  is an additive **probability measure** with **probability distribution**  $\vec{p} = \langle p_i \rangle := \vec{\text{Pl}}$  and additive normalization  $\sum_i p_i = 1$  and operator  $\text{Pr}(A) = \sum_{\omega_i \in A} p_i$ .

$\mathcal{S}$  is **consonant** ( $\mathcal{F}$  is a **nest**) when (without loss of generality for ordering, and letting  $A_0 := \emptyset$ )  $A_{j-1} \subseteq A_j$ . Now  $\Pi(A) := \text{Pl}(A)$  is a **possibility measure** and  $\eta(A) := \text{Bel}(A)$  is a **necessity measure**. Since results for necessity are dual to those of possibility, only possibility will be discussed in the sequel.

As  $\text{Pr}$  is additive, so  $\Pi$  is **maximal**:

$$\forall A, B \subseteq \Omega, \quad \Pi(A \cup B) = \Pi(A) \vee \Pi(B),$$

Table 1: Summary of probability and possibility in GIT.

	RANDOM SET	DISTRIBUTIONS: $i \leftrightarrow j$	
		Probability	Possibility
Focal Set	Any	Singletons: $A_i = \{\omega_i\}$ $\{\omega_i\} = A_i$	Nest: $A_i = \{\omega_1, \dots, \omega_i\}$ $\{\omega_i\} = A_i - A_{i-1}, A_0 := \emptyset$
Structure	None	Partition	Total order
Belief	$\text{Bel}(A) = \sum_{A_j \subseteq A} m_j$	$\text{Pr}(A) := \text{Bel}(A)$	$\eta(A) := \text{Bel}(A)$
Plausibility	$\text{Pl}(A) = \sum_{A_j \cap A \neq \emptyset} m_j$	$\text{Pr}(A) := \text{Pl}(A)$	$\Pi(A) := \text{Pl}(A)$
Relation	$\text{Bel}(A) = 1 - \text{Pl}(\bar{A})$	$\text{Bel}(A) = \text{Pl}(A) = \text{Pr}(A)$	$\eta(A) = 1 - \Pi(\bar{A})$
Distribution	$\text{Pl}_i = \sum_{A_j \ni \omega_i} m_j$	$p_i := \text{Pl}_i = m_i$	$\pi_i := \text{Pl}_i = \sum_{j=i}^n m_j$
Measure		$m_i = p_i$ $\text{Pr}(A \cup B) = \text{Pr}(A) + \text{Pr}(B) - \text{Pr}(A \cap B)$	$m_i = \pi_i - \pi_{i+1}, \pi_{n+1} := 0$ $\Pi(A \cup B) = \text{Pr}(A) \vee \text{Pr}(B)$
Normalization		$\sum_i p_i = 1$	$\bigvee_i \pi_i = 1$
Operator		$\text{Pr}(A) = \sum_{\omega_i \in A} p_i$	$\Pi(A) = \bigvee_{\omega_i \in A} \pi_i$
Nonspecificity	$\sum_j m_j \log_2  A_j $	0	$\sum_{i=2}^n \pi_i \log_2 \left[ \frac{i}{i-1} \right]$ $= \sum_{i=1}^n (\pi_i - \pi_{i+1}) \log_2(i)$
Strife	$-\sum_j m_j \log_2 \left[ \sum_{k=1}^n m_k \frac{ A_j \cap A_k }{ A_j } \right]$	$-\sum_i p_i \log_2(p_i)$	$\sum_{i=2}^n \pi_i - \pi_{i+1} \log_2 \left[ \frac{i^2}{\sum_{j=1}^i \pi_j} \right]$ $< .892$
Semiring	$\langle \oplus, \otimes \rangle$	$\langle +, \times \rangle$	$\langle \vee, \sqcap \rangle$
Marginals		$p(x) = \sum_y p(x, y)$	$\pi(x) = \bigvee_y \pi(x, y)$
Conditionals		$p(x, y) = p(x y) \times p(y)$ $p(x y) = p(x, y)/p(y)$ $\forall y, \sum_x p(x y) = 1$	$\pi(x, y) = \pi(x _{\sqcap y}) \sqcap \pi(y)$ $\pi(x _{\sqcap y}) \in [\pi(x, y), 1]$ $\forall y, \bigvee_x \pi(x _{\sqcap y}) = 1$
Process		$\mathbf{P} := [p(x y)]$ $p' = p \cdot \mathbf{P}$ $p'(x) = \sum_y p(y) \times p(x y)$	$\mathbf{\Pi} := [\pi(x _{\sqcap y})]$ $\pi' = \pi \circ \mathbf{\Pi}$ $\pi'(x) = \bigvee_y \pi(y) \sqcap \pi(x _{\sqcap y})$
Concepts		Division among distinct hypotheses Frequency Chance Likelihood	Coherence around certain hypotheses Capacity Ease of attainment Distance, similarity

where  $\vee$  is the maximum operator. As long as  $C(\mathcal{F}) \neq \emptyset$  (this is required if  $\mathcal{F}$  is a nest), then  $\bar{\pi} = \langle \pi_i \rangle := \bar{\Pi}$  is a **possibility distribution** with maximal normalization  $\bigvee_i \pi_i = 1$  and operator  $\Pi(A) = \bigvee_{\omega_i \in A} \pi_i$ . Also define the core and support of the possibility distribution

$$C(\pi) := \{\omega_i : \pi(\omega_i) = 1\} = C(\mathcal{S}), \quad U(\pi) := \{\omega_i : \pi(\omega_i) > 0\} = U(\mathcal{S}).$$

**Nonspecificity** and **strife** are two **uncertainty measures** which are defined on random sets. They measure respectively the possibilistic and probabilistic aspects of the uncertainty or information represented in the random set. They achieve the forms shown in the table for the possibilistic and probabilistic special cases. Note that in the probabilistic case the uncertainty collapses to stochastic entropy, while in the possibilistic case the strife is bounded above by a small number.

Stochastic and possibilistic processes are defined on their respective distributions when two operators  $\oplus$  and  $\otimes$  are available such that  $\langle \oplus, \otimes \rangle$  form a semiring ( $\otimes$  distributes over  $\oplus$ ), and  $\oplus$  is the operator of the distribution. For probability,  $\oplus = +$ , and  $\langle +, \times \rangle$  is the unique semiring.

For possibility,  $\oplus = \vee$ , and there are many semirings of the form  $\langle \vee, \sqcap \rangle$ , where  $\sqcap$  is a triangular norm (monotonic, associative, commutative operator with identity 1 [5]).  $\wedge$  (the minimum operator) and  $\times$  are two of the more popular norms, as is  $0 \vee (x + y - 1)$ . Conditional possibility is not always unique, depending on the norm used. The formulae for marginal, joint, and conditional probability and possibility (which is dependent on  $\sqcap$ ) are then shown in the table, as is the next state function for a stochastic and possibilistic process.

### 3.2 Possibilistic Models

Probability and possibility almost never coincide (only for distributions of the form  $\langle 0, \dots, 1, \dots, 0 \rangle$ ). Semantically, probability and possibility theory are also related to very different concepts [20]. Probability is inherently additive, and is thus concerned with the dispersal or division of knowledge over a set of distinct hypotheses, and so with concepts related to frequency.

But possibility is inherently *non-additive*. It is concerned with the *coherence* of knowledge *around* a set of certain hypotheses (the core  $C(\pi)$ ), and thus with *ordinal* concepts related to capacity. Where probability makes very strong constraints on the representation of uncertainty (additivity), possibility makes only very *weak* constraints. The maximum relation is a very weak operator, and there is a choice of many norms to use, some of which are strong, and others of which are also weak.

So possibilistic models are appropriate where stochastic concepts and methods are inappropriate, including situations where long-run frequencies are difficult if not impossible to obtain, or where small sample sizes prevail. This is true in **reliability analysis**, for example, where failures and system entry into non-nominal behavior domains are very rare; and **trend-analysis**, where even though observations are made over a long time, the state variables of concern change only very slowly, and new domains of behavior are only very rarely seen. In these cases the weakness of the possibilistic representation is matched by the weak evidence available.

A general modeling relation is shown in Fig. 2, where:  $t, t'$  are former and subsequent times;  $W = \{w\}$  are the states of the world;  $M = \{m\}$  are the states of the model;  $o: W \mapsto M$  is the measurement function;  $r: W \mapsto W$  is the movement of "reality"; and  $f: M \mapsto M$  is the modeling prediction function.

Reality is presumably given, or at least operates on its own without our help. So to make a valid model we are required to provide the measurement and prediction functions so that the diagram commutes. Measurement is used both to set the initial conditions of the model and to corroborate later measurements against the model state, while prediction is used to produce the future model state.

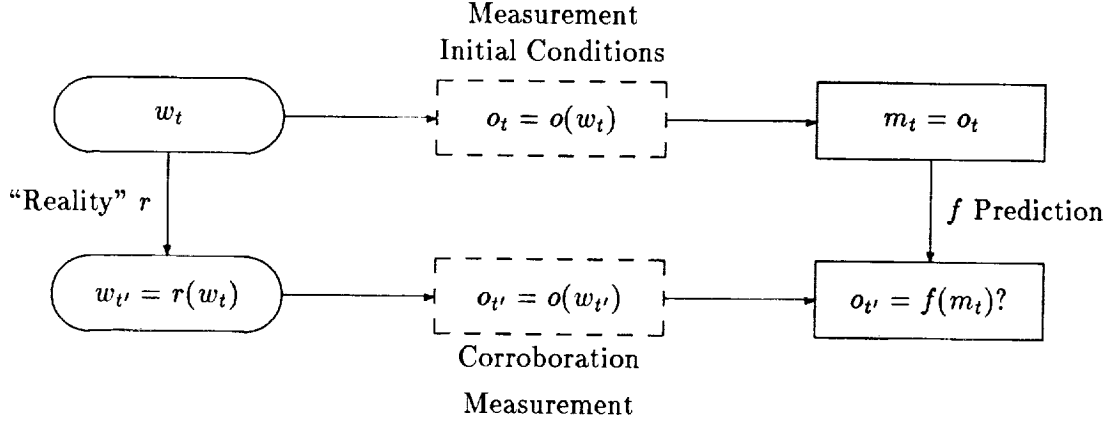


Figure 2: A general modeling relation.

In a stochastic model,  $M = \{\bar{p}\}$ , where  $\bar{p}$  is a probability distribution of the state of the world. And in a possibilistic model  $M = \{\bar{\pi}\}$ , where  $\bar{\pi}$  is a possibility distribution of the state of the world. Once the final time possibility distribution is achieved, then a **possibilistic Monte Carlo method** [24] is required to select a final outcome.

So a possibilistic model requires possibilistic measurement and prediction procedures. Possibilistic prediction procedures will be based on the possibilistic processes briefly outlined in Sec. 3.1. For example, the author has defined **possibilistic automata** as possibilistic Markov processes [22] which generalize non-deterministic automata.

Possibilistic measurement methods have also been developed by the author [18, 21]. The essential requirement is the collection of the frequency of occurrence of subsets or intervals which are partially overlapping. If the core of the observed intervals (their global intersection) is nonempty, then (1) will yield an empirical possibility distribution.

An example is shown in Fig. 3. On the left, four observed intervals are shown. The bottom two occur with frequency 1/2, while each of the upper two have frequency 1/4. Together they determine an empirical random set. The step function on the right is the possibilistic histogram derived from (1).

There are a variety of well-justified continuous approximations of a possibilistic histogram. Two examples are shown in the figure. The rising diagonal on the left is common to both. The two falling continuous curves on the right are distinct to each. The parallelogram form marked  $\pi^*$  is one of the most commonly used continuous approximations, but it must be noted that this is only one possibility among many, including smooth curves. This approach to possibilistic measurement generalizes to  $n$  intervals and to the continuous case.

The core, here  $C(\bar{\pi}) = [1.5, 2]$ , being nonempty, is included in the support  $U(\bar{\pi}) = [1, 4]$ . If the core were empty, then  $\bigvee_i P_i < 1$ , so that  $\bar{P}$  would not be a possibility distribution. In this case, possibilistic normalization procedures, which have also been developed by the author [19], would be required.

## 4 Possibility Theory as a Qualitative Modeling Method

Mathematical possibility, in both theory and applications, is still in the basic research phase, just out of its infancy. For example, the axiomatic basis for possibility theory and the properties of possibility distributions on continuous spaces are still being defined, and the semantics of possibility in physical

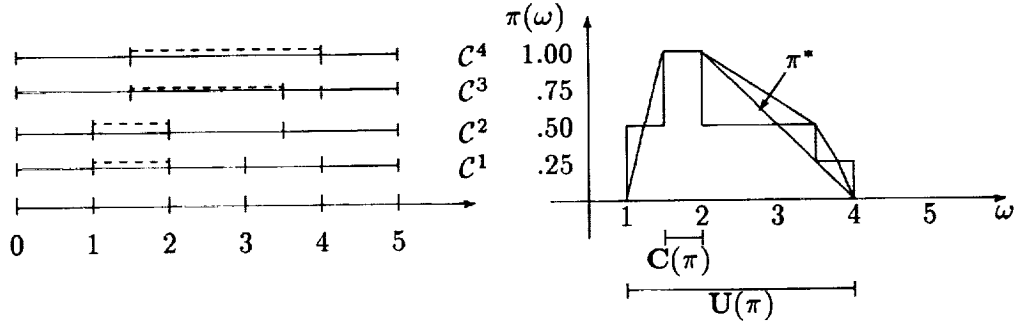


Figure 3: (Left) Four example observed intervals. (Right) The possibilistic histogram and two continuous approximations.

systems has been considered only by very few. But there are many reasons why it can be hoped, and even expected, that possibility theory can come to play an important role in QM in general, and in the application of QM to MBD in particular.

Hamscher et al. have noticed some of the weaknesses of stochastic methods for MBD.

It is usually assumed that reliable failure statistics will be available, but this is in fact rare in practice. What is needed ... is a way of working with likelihoods that could be specified ordinally rather than quantitatively. [17, p. 452]

This is *exactly* what possibility theory provides, a non-additive, ordinal approach to QM which hybridizes interval-valued dynamics and uncertainty distribution methods.

#### 4.1 Possibility Theory and Interval Analysis

Possibility theory can be used as a generalization of interval analysis. A **fuzzy interval** is a convex possibility distribution on  $\mathbb{R}$  where

$$\forall x, y \in \mathbb{R}, \quad \forall z \in [x, y], \quad \pi(z) \geq \pi(x) \wedge \pi(y).$$

This is the case for the measured possibility distributions from Sec. 3.2, as shown in the example in Fig. 3. As illustrated in Fig. 4, under these conditions,  $\pi$  can be represented as a set of nested intervals weighted by their possibility values, where  $\pi^0 := U(\pi)$  as a special case, and

$$\forall \alpha \in (0, 1], \quad \pi^\alpha := \{x \in \mathbb{R} : \pi(x) \geq \alpha\}, \quad \alpha_1 \geq \alpha_2 \rightarrow \pi^{\alpha_1} \subseteq \pi^{\alpha_2}.$$

A standard interval  $[a, b] \subseteq \mathbb{R}$  is a special case, where

$$\pi(x) = \begin{cases} 1, & a \leq x \leq b \\ 0, & x < a \text{ or } x > b \end{cases}, \quad \forall \alpha \in [0, 1], \quad \pi^\alpha = [a, b].$$

For a fuzzy interval  $\pi$  where  $\exists! x \in \mathbb{R}, \pi(x) = 1$ , then  $\pi$  is a **fuzzy number**. **Fuzzy arithmetic** [25] generalizes mathematical operations such as addition and multiplication from interval arithmetic [30] to fuzzy numbers.

#### 4.2 Possibility Theory and Fuzzy Theory

As mentioned above, possibility theory was originally developed by Zadeh [34] in the context of fuzzy sets and fuzzy logic. For Zadeh, a possibility distribution simply *was* a fuzzy set by another name, and



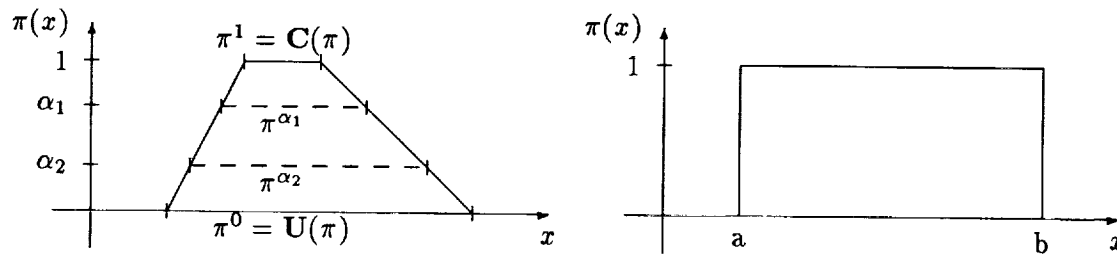


Figure 4: (Left) A possibility distribution as a collection of weighted intervals. (Right) The special case of a crisp interval.

thus possibilistic information theory was *strictly* related to fuzzy information. This view is less than adequate for a variety of reasons. While it is certainly true that a possibility distribution is a fuzzy set, in GIT there are *many* structures which are legitimately fuzzy sets, including *probability* distributions. The author provides a full discussion elsewhere [24].

Nevertheless, fuzzy theory and possibility theory do share a number of points in common. In particular, fuzzy intervals and numbers *are*, in fact, possibility distributions. Thus QM methods which use fuzzy arithmetic as discussed in Sec. 4.1 are essentially possibilistic. An example is the recent work of Sugeno and Takahiro [33].

These methods are also very popular in applications. Fuzzy arithmetic has been used as a QM method for MBD, for example by Shen and Leitch [32] and Fishwick [9]. They use the standard methods of fuzzy control systems, where a set of overlapping fuzzy intervals divide a quantity space into a few linguistic values like “large positive” and “small negative”. These fuzzy sets are not measured properties of the system being modeled, and are dependent on the heuristic specification of the system modeler. Thus they are essentially modeling the cognitive state of some human expert, rather than directly modeling the system in question.

This contrasts sharply with the possibilistic processes discussed in Sec. 3.2. First, they are cast strictly within the context of mathematical possibility theory (including possibilistic processes) specifically, rather than fuzzy theory generally. Also, they are based on measurement of the system in question.

## 5 A Possibilistic Approach to MBD

At both the general level and in some specific ways, there are areas of MBD for which it is appropriate to consider a possibilistic approach.

### 5.1 Possibilistic Symptom and Error Detection

Typically the symptom and error detectors simply compare the measured value against a crisp interval of nominal or predicted values [7]. This is inadequate because the resulting cutoff from nominal to error condition is essentially arbitrary. It is natural to use a fuzzy interval to generalize this, measuring either prediction errors or fault symptoms as the possibilistic distance of the telemetry from the predicted or nominal system state respectively.

Consider a measured value  $x$  compared against an error fuzzy interval of the form of Fig. 4. Such a possibility distribution could be the output of the behavior model, for instance, and would then serve as input to the error detector. Then  $\pi(x)$  is the strength of the error or alarm raised. When  $x \in C(\pi)$ ,

then  $\pi(x) = 1$  and there is no alarm. When  $x \notin U(\pi)$ , then  $\pi(x) = 0$  and the alarm is complete. In between, an intermediate alarm is raised.

Even in situations where crisp thresholds are acceptable, they may be dynamic, varying as a result of changing system and environmental conditions. Doyle et al. consider the situation of an earth-orbiting spacecraft as it proceeds through sunlight and shadow.

Impingent solar radiation changes the thermal profile of the spacecraft, as does the configuration of currently active and consequently, heat-generating subsystems on board. Thresholds on temperature sensors should be adjusted accordingly. A particular temperature value may be indicative of a problem when the spacecraft is in shadow or mostly inactive, but may be within acceptable limits when the spacecraft is in sunlight or many on-board systems are operating. [3]

This situation is shown in Fig. 5. Assume a variable, say the temperature  $t$  of a given component, must be kept in a critical range as the spacecraft moves in and out of daylight. As it does so, the range shifts as shown in the upper figure, where the transition periods begin at a change in sunlight, and continue to thermal equilibrium. For simplicity, assume that that interval is sampled uniformly six times during the orbital day, twice each for daylight  $D_i$ , night  $N_i$ , and transition period  $T_i$ . The possibilistic histogram for the possibility  $\pi(t)$  of  $t$  holding a value at any given time and a parallelogram approximation are shown.

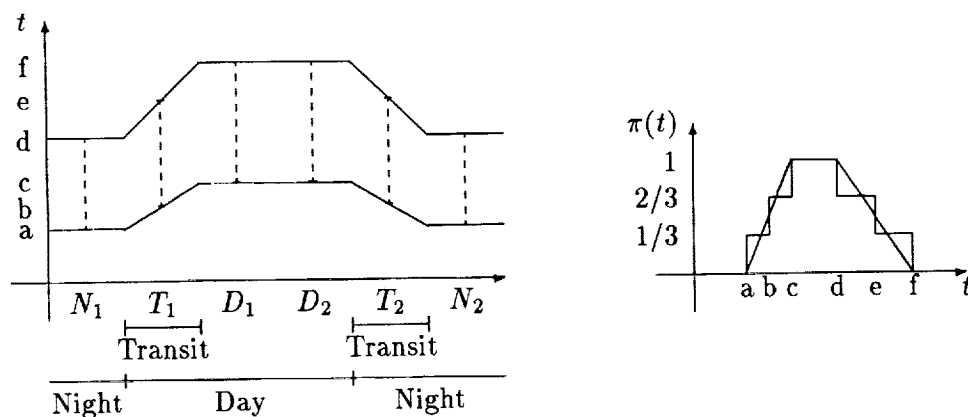


Figure 5: (Left) Variable critical range of a component through a day-night cycle. (Right) Its possibilistic histogram and a parallelogram approximation.

A combination of these two approaches is also possible, where instead of a crisp interval changing over time, rather a whole possibility distribution itself changes with time.

## 5.2 Sensor Modeling

Although the MBD system contains two models, the behavior model and the FGM, as a whole, it is itself also a model of the spacecraft. As such, it is dependent on its inputs from measurement, and thus on the sensor output of the spacecraft. Thus there are modeling issues in MBD concerning the sensors themselves.

When modeling complex systems, sensor data may be sparsely distributed, with missing observations, and sometimes very small samples sizes. As argued elsewhere by the author [20], these are important

conditions for the inapplicability of stochastic methods, and when they hold, possibilistic methods should be considered.

In this respect, there is strong support in the literature for the idea that possibility, as distinct from probability, has a role to play in QM. For example, Luo and Kay observe

When additional information from a sensor becomes available and the number of unknown propositions is large relative to the number of known propositions, an intuitively unsatisfying result of the Bayesian approach is that the probabilities of known propositions become unstable. [29]

While Durrant-Whyte takes a typical statistical approach, he also notes

A robot system uses notably diverse sensors, which often supply only sparse observations that cannot be modeled accurately. [6]

Dvorak and Kuipers make a similar observation in the context of model-based monitoring.

All measurements come from sensors, which can be expensive and/or unreliable and/or invasive. Monitoring is typically based on a small subset of the system parameters, with limited opportunity to probe other parameters. [7]

### 5.2.1 Data Fusion

Possibilistic measurement as outlined in Sec. 3.2 is predicated on the observation of subsets or intervals which are partially overlapping. It is therefore imperative to consider the source of these intervals. But traditional measurement methods do not in general yield overlapping intervals. Rather the purpose in designing a good sensor is to produce distinct outcomes, perhaps intervals with some uncertainty, but still disjoint, forming equivalence classes.

But overlapping intervals *may* result from the combination of data from *different* instruments which measure the same system attribute, either directly or indirectly. Thus random set theory in general and possibility theory in particular is significant when considering the problem of **data fusion** in MBD [15, 29].

Hackett and Shah discuss data fusion in general, including indirect measurements, and the Dempster-Shafer (that is, random set) approach.

Every sensor is sensitive to a different property of the environment; in order to sense multiple properties, it is necessary to use multiple sensors. A system using multiple sensors that sense a single property can be used. [15]

Dubois, Lang, and Prade [4] have also considered the data fusion problem using possibilistic logic.

**Indirect Measurements** First consider the situation where measurements of a component are not made directly, but rather knowledge of the state of the component is only gained indirectly by inference from the outputs of sensors of other components. Doyle et al. [3] offer an example from jet aircraft: low engine thrust can be indicated by either low exhaust temperature or low turbine rotation speed, or both.

This situation is illustrated in Fig. 6. Here component A is not monitored. Its state can only be inferred from the sensors D and E, which monitor components B and C, and which in turn are causally

connected to A. Each of the intervals reported by D and E individually is distinct and disjoint. But since the knowledge of A provided by D and E is mediated by B and C, together they may indicate that A exists in two different, possibly overlapping, intervals.

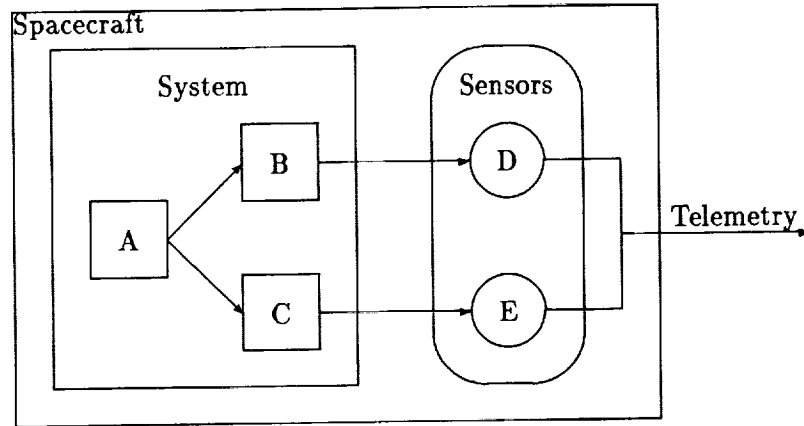


Figure 6: Indirect measurements of the state of a spacecraft component.

So as the amount of sensor “penetration” (sensor/component ratio) drops, standard measurement methods yielding frequency distributions may become less tenable, leaving only observations of random sets.

**Redundant Measurements** Alternatively, a system component may be monitored redundantly by multiple instruments. If these sensors are identical, and identically calibrated, then the result will simply be as if there was a time-series of observations from a single instrument. But if they are mutually discalibrated, either out of phase, or scale, or both, then the intervals reported from each instrument may overlap.

If the sensors measure distinct modalities (e.g. pressure and temperature) of a single component, then a process of **registration** [15] is required to derive a report from one in the modality of the other, or two new reports from each in a third modality. In any event, the argument here is very similar to the one above in the case of indirect measurements, and possibly overlapping intervals may result.

### 5.2.2 Sensor Failure Modeling

As mentioned above, in MBD data are not only combined from disparate sensors, they are also sometimes incomplete, degraded, or missing altogether. Even when standard (disjoint) observations are made, under these conditions there is the potential for the application of GIT and possibility theory.

First, in GIT standard measurements are represented as singleton sets  $\{\omega_i\}$ , where each  $\omega_i \in \Omega$  may indicate a disjoint interval. In a Bayesian or stochastic approach, sensor failure is represented by a uniform distribution over each of the  $\{\omega_i\}$ , again dividing our ignorance among a set of disjoint choices.

But in GIT, a missing observation is represented, more accurately, as an observation of the *entire* universe  $\Omega$ . While this does not result in a specifically possibilistic situation, neither does it result in a frequency or probability distribution. This is discussed more fully by the author elsewhere [24].

For a simple example, assume that a system with three states  $\Omega = \{a, b, c\}$  is observed at ten uniformly distributed times, with  $a$  and  $c$  each seen twice,  $b$  seen three times, and three cases where the sensor made no report. These final three cases must be recorded as observations of  $\Omega$ , and the specific observations

replaced with observations of the singleton sets  $\{a\}$ ,  $\{b\}$ , and  $\{c\}$  respectively. The overall empirical random set is then

$$S = \{\langle\{a\}, 1/5\rangle, \langle\{b\}, 3/10\rangle, \langle\{c\}, 1/5\rangle, \langle\Omega, 3/10\rangle\}$$

with plausibility assignment  $\vec{Pl} = \langle 1/2, 3/5, 1/2 \rangle$ , which is neither an additive probability distribution nor a maximal possibility distribution.

When sensor data are not missing, but rather degraded, compromised, or suspect in some way, a confidence weighting on each sensor's output is naturally not additive: our confidence about the sensors is not divided among them, since all could be perfect or any number of them could be in any state of degradation. Instead it is natural to represent this confidence as a possibility distribution on each sensor's output. Again, an observation in the core indicates complete confidence, while one outside the support indicates complete sensor failure.

Representation of a graduated degree of sensor failure allows a corresponding graduated degree of confidence in model predictions. The need for this has been noted by Fulton.

When we detect a broken sensor, great difficulty arises if we continue diagnosing other failures, because typical rule-based systems do not degrade gently when sensors fail (because the mapping is dependent on a complete and accurate set of sensor data). [11]

### 5.3 Possibilistic Models Proper

In the sequel, the term **system model** will refer to the FGM or the behavior model generally. So finally, it is useful to consider possibilistic methods applied directly to the system models themselves, constructing them as possibilistic processes such as possibilistic automata, and not as fuzzy arithmetic systems as discussed in Sec. 4.

Input to these systems may or may not be proper possibility distributions, since both crisp (standard) intervals and point values are special cases of possibility distributions. But if they are, then it was discussed how telemetry, alarms, and errors can be possibilistically weighted. A possibilistic FGM then would be responsible for producing as its output a set of fault hypotheses which are possibilistically weighted for input to the behavior model. This would in turn generate model prediction errors with possibilistic weights.

A system model which is a possibilistic automata can also be cast as a possibilistic Markov process. As such, its key component is its **transition matrix  $\Pi$** , essentially a vector of conditional possibility distributions as discussed in Sec. 3.1 and shown in Table 3.1. Each conditional possibility distribution represents the possibility, for a given input, of transiting from one system state to another.

The semantics of this transition matrix in a system model is understood in terms of a subsystem-level model where the conditional possibilistic weight indicates a non-additive coupling or relatedness among subsystems. This could be, for example, the efficiency of the subsystem, as in the approach of Doyle et al. [3]. Or, when considering the system model as a causal graph, as in the approach of Hall et al. [16], the weights indicate the degree of causal connectivity between subsystems.

Thus in the possibilistic approach a system model is essentially a possibilistic network, where nonadditive, possibilistic weights are placed on the arcs of a causal graph. The corresponding network appears similar to a Bayesian network, but the mathematics is possibilistic, not stochastic. It has been shown by the author [22] that such nonadditive possibilistic processes are actually the valid generalizations of nondeterministic processes, where stochastic networks are not.

## 6 Conclusion and Future Directions

We have considered possibility theory as a qualitative modeling method in the context of GIT (probability theory, Dempster-Shafer evidence theory, random set theory, and fuzzy theory). We have also defined the key concepts of model-based diagnosis, and have considered, in the context of spacecraft diagnosis, the potential for the application of possibility theory to MBD in terms of symptom and error detection, data fusion, sensor failure modeling, and nonadditive causal graphs.

It should be emphasized that this work is still in the basic research phase. Mathematical possibility theory is still being developed, and most of the key concepts in possibilistic modeling (for example, possibilistic measurement and automata) have only been defined in the past year. The application of possibility theory to the modeling of physical systems and the semantics of possibility in empirical contexts is being considered only by a few.

This work points to many future directions for research, including computer-based implementation of possibilistic models proposed by the author [23], and continued exploration of the conditions for the application of possibilistic modeling to spacecraft systems.

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